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Journal of APPLIED MATHEMATICS AND MECHANICS

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Journal of Applied Mathematics and Mechanics 68 (2004) 475–486

NIKOLAI NIKOLAYEVICH KRASOVSKII (80th birthday tribute)†

Nikolai Nikolayevich Krasovskii is an outstanding Russian scientist who has enriched science with his basic research in a number of fields of mathematics and mechanics. He was born on 7 September 1924 in Ekaterinburg into the family of a doctor. In 1943–1949 he studied at the S. M. Kirov Urals Polytechnic Institute (UPI). There was a cult of mathematics in the UPI at that time, and, by the start of the second course, for Krasovskii the happy opportunity arose of studying mathematics in greater depth under the guidance of the algebraist Professor S. N. Chernikov, Head of the Department of Higher Mathematics. This mainly involved the study of abstract and continuous groups and linear inequalities. The technical nature of the college dictated an interest in the applied side of mathematics and mechanics. After graduating from the Institute, he worked for 10 years in the Department of Higher Mathematics of the UPI as an assistant, associate professor, and head of department. In the period from 1959 to 1970 he worked in the A. M. Gor'kii Urals State University (UrGU), initially as Head of the Department of Theoretical Mechanics, and then as Head of departments that he himself set up – the Department of Calculus, and from 1965 the Department of Applied Mathematics. From 1970 and 1977 he was Head

†*Prikl. Mat. Mekh.* Vol. 68, No. 4, pp. 531–546, 2004.

0021-8928/$—see front matter.

doi: 10.1016/j.jappmathmech.2004.07.001

of the Institute of Mathematics and Mechanics (IMM) of the USSR Academy of Sciences. His scientific authority and his management of the Institute secured its the leading role in the field of mathematics and mechanics in the Urals.

Krasovskii represents the scientific school of N. G. Chetayev, going back to A. M. Lyapunov. Through his first studies on the theory of the stability of motion, carried out under the influence of his teachers Ye. A. Barbashin, N. P. Yerugin, I. G. Malkin and N. G. Chetayev, he became well known. The problem of the stability of the motions of non-linear systems under generalized Routh–Hurwitz conditions, where the region of initial disturbances cannot be considered to be small, was solved, as demanded by engineering practice. The Barbashin–Krasovskii theorem of asymptotic stability was published. Here, the derivative of the Lyapunov function, by virtue of the equations of perturbed motion, may be equal to zero in a set not containing whole trajectories. This theorem received wide recognition and was used repeatedly in theoretical and applied research.

In 1953, he defended his Master's dissertation on the stability of motion under any initial disturbances.

An important problem in the theory of stability is that of the inversion of theorems of Lyapunov's second method. Continuing the work of Ye. A. Barbashin, I. G. Malkin, Kh. L. Massera, K. P. Persidskii and N. G. Chetayev, Krasovskii developed methods for plotting Lyapunov functions, including the solution of the problem of the inversion of the Lyapunov and Chetayev theorems on instability.

In 1955–1957, he worked for his doctorate under N. G. Chetayev. An invaluable gift of fate was the opportunity to see and hear live the greatest mathematicians and engineers of the time and make use of their advice. Successful work towards his doctorate was also aided by the kind consideration of colleagues at the Institute of Mechanics of the USSR Academy of Sciences: G. K. Pozharitsii, V. V. Rumyantsev, B. S. Razumikhin and V. M. Starzhinskii. In 1957 he defended his Doctorate dissertation on some aspects of the theory of stability of non-linear systems.

By the end of the 1950s, he had constructed an original theory of stability for systems with delay. The incentive to carry out this research was provided by the work of A. D. Myshkis and L. E. El'sgol'ts. Krasovskii suggested that elements of functional space of the histories of motion be selected as the phase states of systems with delay. The evolution of such systems can than be described by an ordinary differential equation, but in functional space. This determined the natural theory of stability of hereditary systems using functionals in the role of Lyapunov functions. The universality of this approach was confirmed theorems on the existence of Lyapunov functionals with the necessary properties. The semigroup property of hereditary systems that was revealed in this case determined the explicit sense of the spectral properties of the corresponding infinitesimal operator and opened the way for the transfer to such systems of the theory of critical cases of Lyapunov stability. This approach earned the recognition of specialists and served as the basis for much research.

The results he obtained during this period were published in the monograph *Some Problems of the Theory of Stability of Motion* (1959).

In the 1950s, he began research in the field of optimal control. Interest in this field was aroused by the work of Feldbaum, Buschau, Tsian Sue-Sen and Bellman and in particular by the publication by L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze and Ye. F. Mishchenko of the maximum principle. In 1957, he proposed an approach to problems of optimal control based on methods of functional analysis. A simple and effective theory of linear controlled systems was developed. This approach involved convex programming and focused upon the minimum properties of conjugate designs, including in the treatment of Pontryagin's maximum principle. He then developed a minimax approach to problems of observation under conditions of indeterminate interference. This research was initiated by Kalman.

At the end of the 1950s, Krasovskii had to take upon himself the teaching of courses on the theory of probability in the UrGU and the UPI. This required him to become more thoroughly acquainted with the concepts of Kolmogorov and to study the work of Bernshtein, Khinchin, Feller, Doob Loev, Shannon and Wiener. He became interested in problems of stochastic stability and in problems of the control of random processes, which led to research resulting in the proposal of different approaches to such problems. The combination of classic and new methods made it possible to build a uniform theory of stability and control for ordinary dynamical systems, systems with delay and stochastic systems. The results of Chetayev concerning the stabilization of mechanical systems by forces of a particular nature were extended, and the possibility of constructive improvement in stabilizability based on the gyroscopic effect was substantiated.

Corresponding results of research in the 1950s and 1960s were included in the monograph *The Theory Motion of Control* (1968).

The requirements of actual practice, and knowledge of the work of Isaacs and Bellman, prompted Krasovskii to investigate problems of guaranteed control under conditions of dynamic and information interference. The construction of the theory of differential games, termed positional games, was begun.

Krasovskii put forward the law of extremal aiming, which constructs a control based on the feedback principle by solving auxiliary problems of expected programmed control. This concept was set out in the monograph *The Rendezvous Game Problems* (1970).

A more general concept of the positional differential game was then developed, theorems of the existence of the value of a game and of a saddle point were proved and constructive methods were proposed for generating optimal strategies. The key element was the so-called extremal shift of the real controlled object towards the ideal model, abstract or generated on a computer. The concept was extended to control problems under conditions of conflict and uncertainty of hereditary and stochastic systems, and also systems with degenerate higher derivatives.

The results were presented in the monograph *Positional Differential Games* (1974) (coauthor with A. I. Subbotin).

The properties of irregular equations of the Hamilton–Jacobi type that were revealed in specific problems of positional differential games led to the general concept of the correct solution of such equations. The approach was employed by Subbotin in the form of minimax solutions for a wide class of equations with first-order partial derivatives.

Initially, the constructive solutions were based on auxiliary determinate motions. This limited their effectiveness. By introducing random elements, Krasovskii developed a more effective method which he called stochastic programmed synthesis.

The concept of control under conditions of conflict and uncertainty was a reflection of problems arising in practice and gave these problems an adequate mathematical form. The key concepts were an accessible informational form, the quality index of the process, the minimax-maximin criterion and the control law based on the feedback principle, carrying out a determinate or stochastic strategy. In the latter case, the control realizes outcomes of specially organized random tests. It is fundamental that in this case the optimal result according to the minimax-maximin criterion is achieved with a probability as close as desired to unity.

The control methods and procedures developed were oriented towards state-of-the-art computer technology. A comprehensive computing experiment was specified as a cornerstone element of the investigation and was used without fail to clarify the problem, to search for ways of solving it and for systematically checking hypotheses and results of theoretical investigations. The combination of analytical and computing methods resulted in useful applications.

The results of these investigations were presented in the monograph *Control of a Dynamical System* (1985) and in the monograph *Control under Lack of Information* (1995) (coauthor with A. N. Krasovskii).

Professor Krasovskii is the author of about 300 scientific publications, including six monographs. He promotes the Urals School of Motion Stability Theory and is a founder of the School of the Mathematical Theory of Control. Among his students are engineers and teachers, doctors and masters of science and Corresponding Members and Full Members of the Russian Academy of Sciences.

Working as leader of the IMM of USSR Academy of Sciences, he initiated and encouraged applied studies and the development of the computer section of the Urals Scientific Centre (then the Urals Branch of the USSR Academy of Sciences, and later the Russian Academy of Sciences).

Much of his time and energy is given to reporting on the progress of fundamental science to applied scientists, engineers, teachers, students and schoolchildren. In the 1980s he headed a campaign aiming to computerize schools and colleges in the Sverdlovsk region. This gave impetus to the subsequent raising of standards of schools in Ekaterinburg and the whole region to an adequate level of state-of-the-art information technology.

His authority is high. He has been a member of the Presidium of the Russian Academy of Sciences and a member of the Department of Mechanics and Control Processes of the USSR Academy of Sciences, and he is a member of the Presidium of the National Committee on Theoretical and Applied Mechanics. He is also on the editorial board of leading scientific publications. He has an honorary doctorate from the Urals State Technological University Urals State University, and is a Soros Professor.

His scientific achievements and teaching activity are highly valued by the state (he is a Hero of Socialist Labour, a winner of the Lenin and State prizes and a bearer of the Orders of the Soviet Union and Russia) and by the scientific community (he was awarded the M. V. Lomonosov Great Gold Medal of the Russian Academy of Sciences, the A. M. Lyapunov Gold Medal, the Demidov Prize in physics and mathematics and the 'Triumph' Prize; he is also a doctor honoris causa of the Hungarian Academy of Sciences and has received an IEEE award).

The editorial board and staff of *Prikladnaya Matematika i Mekhanika* and his many disciples, colleagues and friends send him warm greetings on his birthday and wish him good health and further success in his work.

LIST OF THE PRINCIPAL SCIENTIFIC PUBLICATIONS OF N. N. KRASOVSKII

1952

Theorems on the stability of motions governed by a system of two equations. *Prikl. Mat. Mekh.*, 1952, **16**, 547–554.

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1953

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1956

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1961

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Translated by P.S.C.